J. Physiol. (1986), **373**, *pp.* 235–242
With 3 *text-figures*
Printed in Great Britain

THE DETERMINANTS OF THE STEP FREQUENCY IN WALKING IN HUMANS

BY G. A. CAVAGNA AND P. FRANZETTI

From the Istituto di Fisiologia Umana dell'Università di Milano, and the Centro di Studio per la Fisiologia del Lavoro Muscolare del C.N.R., 20133 Milano, Italy

(*Received 12 July 1985*)

SUMMARY

1. The mechanical power spent during walking in lifting and accelerating the centre of mass, $\dot{W}_{ext}$, has been measured at three given speeds maintained at different step frequencies: at any given speed, $\dot{W}_{ext}$ is smaller the greater the step frequency used.

2. The mechanical power spent in accelerating the limbs relative to the centre of mass during walking at a given speed, but with different step frequencies, $\dot{W}_{int}$, was calculated from previous data obtained during free walking (Cavagna & Kaneko, 1977). At a given walking speed, $\dot{W}_{int}$ increases with the step frequency.

3. The total power, $\dot{W}_{tot} = \dot{W}_{ext} + \dot{W}_{int}$, reaches a minimum at a step frequency which is 20–30 % less than the step frequency freely chosen at the same speed.

4. The step frequency at which $\dot{W}_{tot}$ is minimum increases with speed in a similar way to the natural step frequency during free walking.

INTRODUCTION

General features of terrestrial locomotion are the braking action of the ground and the necessity to 'reset' the limbs at each step: when a given speed is maintained with longer steps at a lower frequency the impact against the ground is increased, whereas when the same speed is maintained with shorter steps at a greater frequency the limbs must be accelerated backwards and forwards more times per minute.

The aim of this paper is to determine how these two contrasting effects of a frequency change affect the mechanical power output in walking, and to define the relationship between mechanical power and step frequency at a given speed. To this purpose the work done during walking can be conveniently divided into two parts, as originally proposed by Fenn (1930*a*, *b*) for running: the work necessary to lift and accelerate the centre of mass at each step, W_{ext}, and the work necessary to accelerate the limbs relative to the centre of mass, W_{int}. In fact, the interaction between the moving body and the ground causes vertical displacements and forward speed changes of the centre of mass (as in a 'square wheel'); these can be reduced by decreasing the step length, thus approaching an angle of 90 deg between the forward velocity vector and the link between the centre of mass and the ground (as in a wheel). However, a reduction of the step length, at a given speed, implies an increase in the step frequency and, as a consequence, an increase in the work done in unit time in accelerating the limbs relative to the centre of mass.

The existence of an 'optimum' step frequency was therefore tested by measuring $\dot{W}_{\text{ext}}$, $\dot{W}_{\text{int}}$ and the total power, $\dot{W}_{\text{tot}} = |\dot{W}_{\text{ext}}| + |\dot{W}_{\text{int}}|$ (assuming no energy transfer between $\dot{W}_{\text{ext}}$ and $\dot{W}_{\text{int}}$) at three given speeds, maintained with different step frequencies. Some of the results have already been reported briefly (Cavagna & Franzetti, 1982).

METHODS

The mechanical work done during each step in lifting and accelerating the centre of mass of the body in the forward and vertical directions, W_{ext}, was measured by means of a strain-gauge platform ($4 \times 0{\cdot}5$ m) sensitive to the vertical and the horizontal components of the force exerted by the feet against the ground. The apparatus, and the procedure used to calculate the mechanical energy changes of the centre of mass of the body from the platform's records have been described in detail by Cavagna (1975) and by Cavagna, Franzetti & Fuchimoto (1983). At the end of each run over the platform, the mechanical energy changes of the centre of mass (gravitational potential and kinetic energy) were plotted automatically as a function of time using a microcomputer (Fig. 1). Each of the three sets of tracings in Fig. 1 refer to steps of different length and frequency at the walking speed of about 6·5 km h^{-1}. In each set of tracings the upper curve indicates the changes in kinetic energy of forward motion, $E_{\text{k,f}} = \frac{1}{2}(MV_{\text{f}}^2)$ where M is the body mass and V_{f} the instantaneous forward velocity of the centre of mass. The dotted line in the middle tracing indicates the change in potential energy, E_{p}, due to the vertical displacement of the centre of mass; the continuous line in the middle tracing indicates the gravitational potential energy plus the kinetic energy of vertical motion ($E_{\text{p}} + E_{\text{k,v}}$ where $E_{\text{k,v}} = \frac{1}{2}(MV_{\text{v}}^2)$, V_{v} being the instantaneous velocity of the centre of mass in the vertical direction). The lower curves give the total mechanical energy $E_{\text{tot}} = E_{\text{k,f}} + E_{\text{p}} + E_{\text{k,v}}$ (continuous line) and the sum $E_{\text{k,f}} + E_{\text{p}}$ (dotted line). The increments of $E_{\text{k,f}}$ represent the positive work necessary to accelerate forward the centre of mass during each step (W_{f}), the increments of E_{p} the positive work against gravity (W_{v}) and the increments of E_{tot} the mechanical work done by the muscles against the external forces to increase the mechanical energy of the centre of mass (W_{ext}). The positive work done during each step (W_{f}, W_{v} and W_{ext}) was calculated by the microcomputer from the increments of the curves. The mechanical power output ($\dot{W}_{\text{ext}}$) was obtained by multiplying the positive work done in one step by the step frequency.

$\dot{W}_{\text{ext}}$ was measured in six male subjects whose characteristics are given in Table 1. The subjects, all wearing gym shoes, walked in a corridor at three given speeds (following a mark pulled by a motor at 4·6, 5·5 and 6·6 km h^{-1}) and with different step frequencies (dictated by a metronome). The subjects were able to meet the motor's velocity with the accuracy indicated in Table 2.

Changing the step frequency far from the natural one may lead to drastic modifications of the mechanics of walking; for example, high step frequencies (and short step lengths) may naturally lead to the mechanics of running, in spite of the recommendation made to the subject to maintain the mechanics of walking. Fortunately there is a way of assessing whether the mechanics of walking are maintained or not. This is represented by

$$\% \text{ recovery} = ((W_{\text{f}} + W_{\text{v}} - W_{\text{ext}})/(W_{\text{f}} + W_{\text{v}})) \times 100,$$

which gives the amount of transfer between work done by gravity, W_{v}, into work in accelerating forwards, W_{f}, and vice versa. This transfer would be 100 % in a frictionless pendulum; it is 0–4 % at all speeds of running and reaches about 65 % at intermediate speeds of walking (Cavagna, Thys & Zamboni, 1976). All the data given in Fig. 2 refer to experiments in which the percentage recovery was within ± 5 % of the value indicated by the dotted curve in Fig. 4 of Cavagna *et al.* (1983) which is based on measurements made in seventeen adult subjects during free walking. At a given speed, the % recovery tends to increase when the step length is increased above the natural one and to decrease when the step length is decreased below it.

$\dot{W}_{\text{int}}$ was calculated from 'displacement curves' (Fenn, 1930*a*) giving the orientation, during a free walking step, of the limbs (upper arm, forearm, thigh and lower leg) of four subjects studied in a previous paper (Cavagna & Kaneko, 1977); one of those subjects (G.C.) also served in the present experiments measuring $\dot{W}_{\text{ext}}$ (Table 1). All (twenty-five) of the displacement curves made by Cavagna & Kaneko (1977) were used; these curves refer to step lengths, L, ranging between 0·64 and 1·36 m. From each original step length, a period of the step $\tau = L/\bar{V}_{\text{f}}$ was calculated, as if the steps were taken at $\bar{V}_{\text{f}} = 4{\cdot}6$, 5·5 and 6·6 km h^{-1}. The period so obtained, was considered the

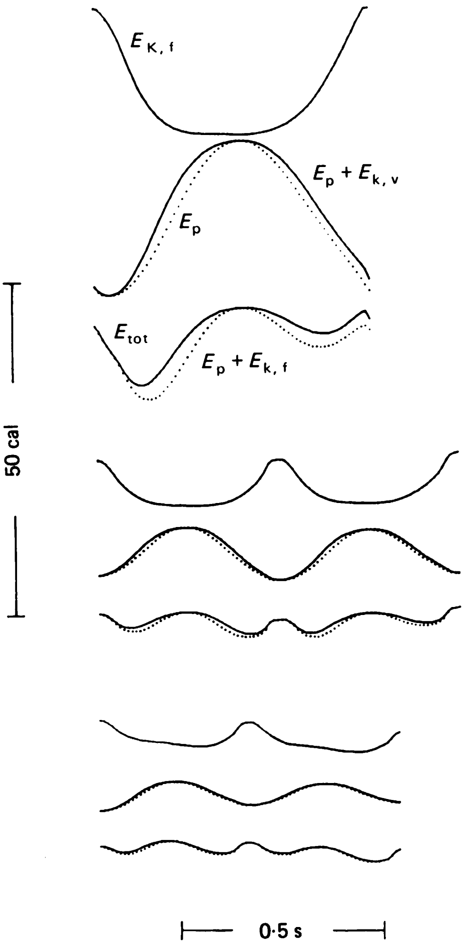

Fig. 1. Computer plots of the mechanical energy changes of the centre of mass of the body during three steps of different length and frequency at a walking speed of about 6·5 km h^{-1} (subject G.C.). In each set of tracings the upper curve refers to the kinetic energy of forward motion, $E_{k,f}$, the middle curve to the sum of the gravitational potential energy, E_p, and of the kinetic energy of vertical motion, $E_{k,v}$, and the bottom curve to the total energy, $E_{tot} = E_{k,f} + E_p + E_{k,v}$. The curves E_p and $E_{k,f} + E_p$ are also given (dotted lines). In the upper set of tracings: the exact speed of walking, $\bar{V}_f = 6{\cdot}57$ km h^{-1}, the power, $\dot{W}_{ext} = 16{\cdot}3$ cal kg^{-1} min^{-1}, the step length, $L = 1{\cdot}24$ m, the step frequency, $f = 88$ steps min^{-1} and the % recovery, $R = 65{\cdot}7$ %. In the middle set of tracings: $\bar{V}_f = 6{\cdot}34$ km h^{-1}, $\dot{W}_{ext} = 8{\cdot}6$ cal kg^{-1} min^{-1}, $L = 0{\cdot}78$ m, $f = 135$ steps min^{-1} and $R = 65{\cdot}8$ %. In the bottom set of tracings: $\bar{V}_f = 6{\cdot}43$ km h^{-1}, $\dot{W}_{ext} = 6{\cdot}0$ cal kg^{-1} min^{-1}, $L = 0{\cdot}66$ m, $f = 162$ steps min^{-1} and $R = 60{\cdot}0$ %.

new abscissa of the original displacement curve. $\dot{W}_{int}$ was then calculated from the displacement curves on the basis of this new abscissa and plotted, for a given speed, as a function of the corresponding frequency value $f = 1/\tau$. Each of the three dotted lines in Fig. 2*B* was calculated from the twenty-five values of $\dot{W}_{int}$ and f obtained in this way; the three dotted lines in Fig. 2*A*, on the contrary, were calculated using only the five displacement curves determined by Cavagna & Kaneko (1977) on subject G.C.

The procedure described above implies that the displacement of the limbs during one step is

TABLE 1. Characteristics of the subjects

Subject	Age (yr)	Weight (kg)	Height (m)	Leg length (m)
		$\dot{W}_{ext}$ measurements		
G.C.	50	79·5	1·77	0·90
T.F.	25	64·0	1·72	0·87
P.F.	24	72·0	1·75	0·90
M.P.	22	65·0	1·72	—
A.B.	23	62·0	1·74	0·88
E.T.	21	80·5	1·78	0·90
Mean	27	70·5	1·75	0·89
		$\dot{W}_{int}$ measurements		
G.C.	39	78·0	1·77	0·90
M.S.	22	56·0	1·75	0·89
P.C.	23	77·0	1·77	0·89
A.Z.	29	70·0	1·78	0·91
Mean	28	70·2	1·77	0·90

TABLE 2. Speeds of walking during $\dot{W}_{ext}$ measurements

Subject	Speed 1	Speed 2 (mean ± S.D. (n), km h^{-1})	Speed 3
G.C.	4·64 ± 0·15 (11)	5·50 ± 0·19 (6)	6·53 ± 0·18 (12)
T.F.	4·62 ± 0·09 (5)	5·53 ± 0·16 (4)	6·74 ± 0·12 (5)
F.P.	4·69 ± 0·07 (3)	5·48 ± 0·12 (6)	6·58 ± 0·07 (4)
M.P.	4·49 ± 0·04 (4)	5·40 ± 0·21 (4)	6·56 ± 0·09 (3)
A.B.	4·62 ± 0·14 (5)	5·55 ± 0·09 (13)	6·63 ± 0·16 (10)
E.T.	4·58 ± 0·12 (9)	5·48 ± 0·15 (15)	6·62 ± 0·19 (12)
Mean	4·61 ± 0·13 (37)	5·50 ± 0·14 (48)	6·61 ± 0·17 (46)

uniquely determined by the step length, independently of the step duration. This assumption seems reasonable for the lower limbs which, with their movements, contribute to most (80–90 %) of W_{int} (Cavagna & Kaneko, 1977).

$\dot{W}_{int}$ was calculated assuming a complete transfer of kinetic energy between the two segments of each limb (Cavagna & Kaneko, 1977). This minimum value of $\dot{W}_{int}$ has been used to compensate for a possible error made by assuming no transfer between $\dot{W}_{int}$ and $\dot{W}_{ext}$ in the calculation of $\dot{W}_{tot}$ which, as mentioned above, is taken as the sum of the absolute values of $\dot{W}_{int}$ and $\dot{W}_{ext}$.

The estimating equation used to define the correlation between mechanical power and step frequency was

$$\dot{W}(\text{cal kg}^{-1}\ \text{min}^{-1}) = af^{b}(\text{steps min}^{-1}). \quad (1)$$

The values of the constants a and b of eqn. (1) were calculated by the least-squares method.

For $\dot{W}_{int}$ (dotted lines in Fig. 2) the values of the constants are, in Fig. 2*A* (subject G.C. only): $a = 4{\cdot}2 \times 10^{-5}$ and $b = 2{\cdot}66$ ($r = 0{\cdot}95$, $n = 5$) at 4·6 km h^{-1}; $a = 4{\cdot}1 \times 10^{-5}$ and $b = 2{\cdot}68$ ($r = 0{\cdot}95$, $n = 5$) at 5·5 km h^{-1}; and $a = 4{\cdot}3 \times 10^{-5}$ and $b = 2{\cdot}68$ ($r = 0{\cdot}95$, $n = 5$) at 6·5 km h^{-1}. In Fig. 2*B* (all four subjects studied by Cavagna & Kaneko, 1977, including G.C.): $a = 3{\cdot}7 \times 10^{-4}$ and $b = 2{\cdot}12$ ($r = 0{\cdot}91$, $n = 25$) at 4·6 km h^{-1}; $a = 4{\cdot}2 \times 10^{-4}$ and $b = 2{\cdot}12$ ($r = 0{\cdot}91$, $n = 25$) at 5·5 km h^{-1}; and $a = 4{\cdot}9 \times 10^{-4}$ and $b = 2{\cdot}13$ ($r = 0{\cdot}91$, $n = 25$) at 6·6 km h^{-1}.

For $\dot{W}_{ext}$ (continuous lines in Fig. 2) the values of the constants are, in Fig. 2*A* (subject G.C.): $a = 1322$ and $b = -1{\cdot}160$ ($r = 0{\cdot}93$, $n = 11$) at 4·6 km h^{-1}; $a = 10054$ and $b = -1{\cdot}536$ ($r = 0{\cdot}95$, $n = 6$) at 5·5 km h^{-1}; and $a = 33484$ and $b = -1{\cdot}693$ ($r = 0{\cdot}91$, $n = 12$) at 6·5 km h^{-1}. In Fig. 2*B* (all six subjects of Table 1): $a = 543$ and $b = -0{\cdot}977$ ($r = 0{\cdot}79$, $n = 37$) at 4·6 km h^{-1}; $a = 2510$ and $b = -1{\cdot}237$ ($r = 0{\cdot}90$, $n = 48$) at 5·5 km h^{-1}; and $a = 16587$ and $b = -1{\cdot}546$ ($r = 0{\cdot}89$, $n = 46$) at 6·6 km h^{-1}.

The estimating equation used to define the correlation between step frequency and free walking speed was of the same kind as eqn. (1)

$$f(\text{steps min}^{-1}) = c\,\bar{V}_f^{\,d}(\text{km h}^{-1}). \quad (2)$$

The values of the constants of eqn. (2) were calculated by the least-squares method from data obtained on the nine subjects of Table 1 during normal walking (Cavagna *et al.* 1976; Cavagna & Kaneko, 1977; Cavagna *et al.* 1983 and this study). For subject G.C. (Fig. 2*A*) the values of the constants are: $c = 40{\cdot}84$ and $d = 0{\cdot}604$ ($r = 0{\cdot}99$, $n = 40$). For all the subjects used to measure $\dot{W}_{int}$ and $\dot{W}_{ext}$ (Figs. 2*B* and 3) the constants are: $c = 44{\cdot}93$ and $d = 0{\cdot}572$ ($r = 0{\cdot}98$, $n = 166$).

RESULTS

The power output due to the kinetic and gravitational potential energy changes of the centre of mass of the body during walking, $\dot{W}_{ext}$, is given as a function of the step frequency in Fig. 2 (data points and continuous lines). It can be seen that, at

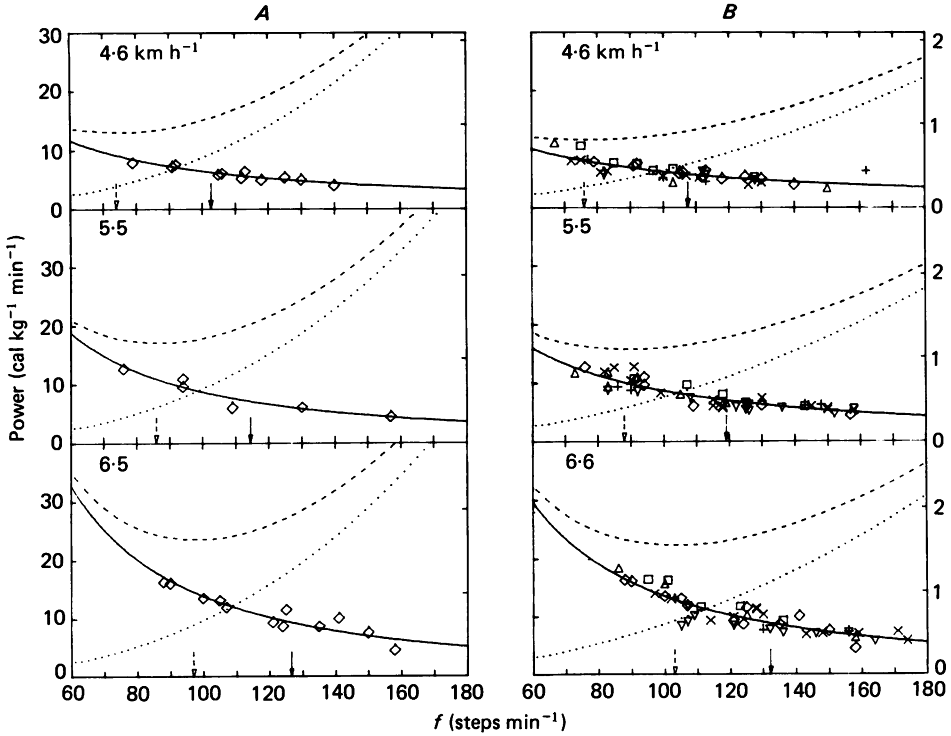

Fig. 2. The weight-specific mechanical power output during walking at the indicated speeds is given in each set of tracings as a function of the step frequency. The data points and the continuous line through them give the power output due to the kinetic and gravitational potential energy changes of the centre of mass of the body ($\dot{W}_{ext}$), the dotted line indicates the power required to accelerate the limbs relative to the centre of mass ($\dot{W}_{int}$) and the dashed line is the sum of the two ($\dot{W}_{tot}$). A minimum of $\dot{W}_{tot}$ is attained at the frequency indicated on the abscissa by the interrupted arrow; the continuous arrow gives the natural frequency, freely chosen during walking at the indicated speeds. *A*: $\dot{W}_{ext}$, $\dot{W}_{int}$ and the natural frequency were all measured on one subject (G.C.). *B*: $\dot{W}_{ext}$ and $\dot{W}_{int}$ were measured on different subjects (except for G.C.); the natural frequency was calculated from the data obtained in all of them. The symbols refer to different subjects as follows: ◇, G.C.; □, T.F.; +, M.P.; △, P.F.; ▽, A.B.; ×, E.T. (see Table 1).

a given speed, $\dot{W}_{ext}$ decreases when the step frequency is artificially increased above its natural value (calculated from eqn. (2) and indicated by the continuous arrow on the abscissa), whereas it increases when the step frequency is decreased below its natural value. The change is greater the greater the speed. On the contrary, the power necessary to accelerate the limbs relative to the centre of mass at a given walking speed increases with the step frequency (dotted lines).

The dashed lines in Fig. 2 indicate the total mechanical power output ($\dot{W}_{tot} = \dot{W}_{ext} + \dot{W}_{int}$) and were obtained by summing the continuous and the dotted lines. The dashed lines show that the total power indeed reaches a minimum at intermediate frequencies; the minimum occurs at the step frequency indicated on the abscissa by the interrupted arrow. This calculated step frequency is 20–30 % less than the natural one (continuous arrows) and increases similarly with speed (see also Fig. 3). This is true both for the case illustrated by Fig. 2*A* in which external power, internal power and natural step frequency were all measured in the same subject, and for Fig. 2*B* which gives an average of the results obtained in separate studies on different subjects.

The step frequency at which the total power is minimum is plotted as a function of the speed of walking in Fig. 3 for comparison with the natural step frequency (continuous line). Fig. 3 shows clearly that the calculated step frequency increases with the walking speed with about the same slope as the natural one. In addition, Fig. 3 shows how the results change when the W_{ext} data accepted for calculations include those with percentage recovery within $\pm 10\%$ ($\triangle$) and $\pm 15\%$ (+) of the normal value (see Methods).

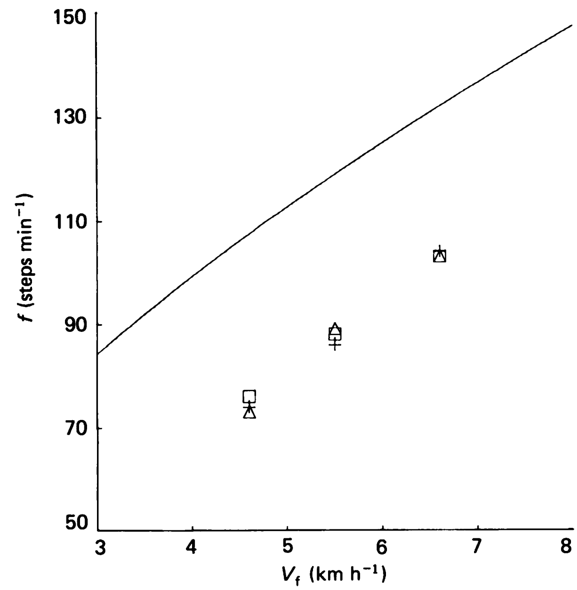

Fig. 3. The continuous line indicates the relationship between natural step frequency during free walking (ordinate) and walking speed (abscissa). □, calculated frequency values indicated by the interrupted arrows in Fig. 2*B*; these values refer to $\dot{W}_{ext}$ curves calculated from data with percentage recovery within 5 % of the normal value (see text). Inclusion of the data with percentage recovery within ± 10 and $\pm 15\%$ of the normal modifies the results as indicated by $\triangle$ and + respectively.

DISCUSSION

The idea that movements are performed according to a principle of minimum energy output is broadly accepted in physiology. In walking, Magne (1920) and Zarrugh & Radcliffe (1978) found that the freely chosen step rate requires the least oxygen consumption at any given speed. This finding cannot be explained solely by a minimum of the mechanical work done by the muscles; also the efficiency of muscular contraction must be taken into account, and the efficiency is known to change with the speed of shortening and therefore with the frequency of the movements.

Some features of normal walking were successfully predicted by mathematical models minimizing the mechanical work done by the individual muscular groups at the level of the joints of the lower limbs (Beckett & Chang, 1968; Chow & Jacobson, 1971). In addition, the time of swing in walking calculated according to a purely ballistic model, without muscular contractions, was found to be similar to the times observed experimentally in adults during free walking (Mochon & McMahon, 1980). If the oscillation of the swing leg were mainly a passive pendular motion, then the power actually spent by the muscles in accelerating the limbs would be less than that indicated by the dotted lines in Fig. 2 and the calculated step frequency would approach the natural one.

In conclusion, several studies as well as the present one indicate that both the metabolic and mechanical energy output during walking are at a minimum when the step frequency used is similar to the natural one. However, how does the subject sense the metabolic and the mechanical energy output? Control of the step frequency on the basis of the energy output would require learning or complicated and unknown sensing mechanisms. Forces, more than metabolic or mechanical energy output, may be more easily sensed and kept to a minimum in the control of the step frequency in walking. One could argue that other requirements, instead of a minimization of forces, may be considered in the control of step frequency: e.g. double support time and stability, the possibility of foot slip at heel strike and foot clearance during swing. However, it is not clear how taking care of these requirements should imply a minimum of energy expenditure, whereas it is likely that a minimum average force may involve a minimum of mechanical work done and, as a consequence, a reduced metabolic energy expenditure.

An illuminating example is offered by the respiratory apparatus: the respiratory frequency freely chosen at a given alveolar ventilation has also been found to be near a frequency at which the mechanical power of breathing is minimal (Otis, Fenn & Rahn, 1950); however, it was found subsequently that the respiratory frequency was even more accurately predicted by a minimum of the average force exerted by the respiratory muscles than by a minimum of power (Meade, 1960). The same could be true for walking: the impact against the ground (at a lower step frequency) and the stiffening of the limbs (at a greater step frequency) may be sensed and reduced to a minimum during free walking.

This work was supported in part by the Ministry of Education of Italy.

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